

## Mathematics: How to solve quadratic equations (13-14 years)

### Subject

The students learn two strategies to solve simple quadratic equations:

1. Reduce the equation to  $X^2 = \text{number}$
2. Reduce the equation to  $A \cdot B = 0$  so  $A=0$  v  $B=0$  give the solutions.

They learn to use factorization (not yet the a,b,c - formula).

Duration: 3 - 4 lessons.

Objective: obtaining algebraic skills.

### Regular lessons (not differentiated)

The theory in the textbook is divided, build up in small steps. You have to complete all the assignments of one step before you can start with the next one. Only after completing all the steps, the textbook offers mixed assignments referring to the whole chapter. (Behavioristic perspective)

### Differentiated lessons (after reversal and omission)

The students that choose for this option are allowed to work in teams outside the classroom. They receive an instructionsheet with worked-out examples for solving all kinds of quadratic equations. In addition, they get a stack of cards on which are three equations to solve. There are cards in five different levels. The students are free to choose which assignments are made and can check their results on an answer sheet. They know the skill of level 3 corresponds to "sufficient", level 4 is "good" and level 5 exceeds the level of the final test all the students have to make at the end of the Chapter. By solving the equations from the cards, the students practise the different methods and at the same time they learn to decide which method to use in which situation (heuristics).

### Experiences

To put it mildly: practising algebraic skills is not the most favorite activity for the more talented students. They think it is boring, too much hassle and totally unnecessary: "I solved the problem so why bother?"

By working with the cards, practicing skills is more fun and more motivating. The students experience the usefulness of the skills and are challenged to become good at it. There is a direct reward: being proud of yourself when you accomplish a higher level. (gamification)

For me as their teacher it was not necessary to supervise the group being at work outside the classroom. Of course I paid them a visit once or twice, but that was to show interest in how they were doing and to offer some help. In the meantime you can pay more attention to the rest of your group in the classroom, because this group is significantly smaller now.

## Method 2: Product = 0

### Approach:

You reduce a quadratic equation to a product that equals zero.

You start by moving everything from the right side to the left side of the equation, so the right hand becomes zero. Then you factorise the left side and you put all separate factors equal to zero.

This method will be new for you, but is easy to understand: when you multiply two numbers and the outcome gives zero, then (at least) one of the numbers has to be zero. So the rule you use is:  $A \cdot B = 0 \rightarrow A = 0 \vee B = 0$ .

### Examples:

$3x(x + 2) = 0$ <i>use product = 0</i> $3x = 0 \vee x + 2 = 0$ $x = 0 \vee x = -2$	$x^2 + 3x = 5x$ <i>make right side 0</i> $x^2 - 2x = 0$ <i>factorise left side</i> $x(x - 2) = 0$ <i>use product = 0</i> $x = 0 \vee x - 2 = 0$ $x = 0 \vee x = 2$	$x^2 + 3x = 10$ <i>make right side 0</i> $x^2 + 3x - 10 = 0$ <i>factorise left side</i> $(x + 5)(x - 2) = 0$ <i>use product = 0</i> $x + 5 = 0 \vee x - 2 = 0$ $x = -5 \vee x = 2$
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Of course you can always use your textbook for further explanations. Now that you have become acquainted with the two methods you can start to practise them. After a while you become fast and handy. You will learn to recognize which method has to be used in all kinds of different equations. And after a while you even discover that sometimes you can choose between the two methods!

You see?

	method 1	method 2
	$x^2 - 25 = 0$ <i>square = number</i>	$x^2 - 25 = 0$ <i>factorise left side</i>
	$x^2 = 25$ <i>+ and - root</i>	$(x - 5)(x + 5) = 0$ <i>use product = 0</i>
	$x = 5 \vee x = -5$	$x = 5 \vee x = -5$

To make practising a bit less boring you now take the stack of cards. On each card you find three equations to solve. There are cards in five different levels. Choose a card. Write the equations in your notebook and solve them. Check if you have the correct answers. Learn from your mistakes! When you are confident you take a card from a higher level. Which level can you reach?

**Good luck!!**

1.  $x^2 - 6x = 0$

2.  $6x^2 - 5 = 1$

3.  $x^2 + 7x + 10 = 0$

Level 1

card ①

1.  $x^2 + 5x - 24 = 0$

2.  $x^2 = 9$

3.  $x^2 + 10x = 0$

Level 2

card ②

1.  $x^2 - 25 = 0$

2.  $x^2 - 5x + 6 = 0$

3.  $3x^2 - x = 0$

Level 3

card ③

1.  $x^2 - 6x + 8 = 0$
2.  $5x^2 + 4x = 0$
3.  $5x^2 + 20 = 20$

Level I

card ④

1.  $x^2 + x = 9x$
2.  $9 = x^2 - 1$
3.  $x^2 + 9x = 10$

Level II

card ①

1.  $5x^2 + 2 = 0$
2.  $x^2 - 10 = 3x$
3.  $x^2 + 10x = 2x$

Level II

card ②



1.  $x^2 - 6x + 8 = 0$

2.  $5x^2 + 4x = 0$

3.  $5x^2 + 20 = 20$

Level I card ④

1.  $x^2 + x = 9x$

2.  $9 = x^2 - 1$

3.  $x^2 + 9x = 10$

Level II card ①

1.  $5x^2 + 2 = 0$

2.  $x^2 - 10 = 3x$

3.  $x^2 + 10x = 2x$

Level II card ②

1.  $10x - 25 = x^2$

2.  $25 - x^2 = 9$

3.  $x^2 + x = 2x$

Level II card ③

1.  $9x = x^2 - x$

2.  $x^2 - 3 = 22$

3.  $x^2 + 11x = -10$

Level II card ④

1.  $x(x + 2) = 9x$

2.  $(2x + 8)(3x - 6) = 0$

3.  $(2x + 8)(3x - 6) = 24x$

Level III card ①

1.  $(x - 5)^2 = 16$
2.  $(x - 5)^2 = 16x$
3.  $(x - 8)(2x - 6)(x + 5) = 0$

Level III

card ②

1.  $3(x^2 - 1) = 3$
2.  $(x - 1)^2 = 40 - (x + 3)^2$
3.  $0,6x^2 - (x - 6) = 3x(0,2x - 1)$

Level III

card ③

1.  $(5x - 7)(4x + 32) = 0$
2.  $5x^2 + 4x = 3x(x - 7)$
3.  $0,25x^2 - 1 = 15$

Level III

card ④

1.  $x(x + 2) = 1 + x^2$
2.  $20 + x(x - 12) = 4(2 - x)$
3.  $-(2x - 1)^2 = 2x - 1$

Level III

card ⑤

1.  $2(x^2 + 15x) - 18 = 50$
2.  $(x^2 - 16)(x^2 - 9) = 0$
3.  $\frac{1}{2}x \left(x - \frac{1}{3}\right) + \frac{1}{3}x = \frac{1}{6}x \left(x - \frac{1}{2}\right)$

Level IV

card ①

1.  $\frac{3}{4}x^2 + \frac{2}{3}x = \frac{1}{6}x + \frac{1}{2}x^2$
2.  $3x^3 - 27x^2 = 156x$
3.  $(2x - 1)^2 = 16$

Level IV

card ②



1.  $4x^3 - 8x^2 - 32x = 0$

2.  $\frac{1}{5}x^2 + 5x = 2\frac{1}{2}x$

3.  $3(x^2 - 5x) - 9 = 2x^2 + x + 48$

Level IV

card ③

1.  $(x^2 - 4)(x + 6) = 0$

2.  $0,2(x - 3)^2 + 1,9 = 9,1$

3.  $2(x - 1)^2 - (x - 5)^2 = 17$

Level IV

card ④

1.  $-\frac{2}{5}(x^2 + 4x) = 4(\frac{1}{2} - \frac{2}{5}x)$

2.  $x^3 - 2x^2 - 48x = 0$

3.  $(4x + 2)^2 = 36$

Level IV

card ⑤

1.  $x^3 = 9x$

2.  $x^8 - 17x^4 + 16 = 0$

3.  $2 - \left(\frac{1}{2}x + 3\right)^2 = -47$

Level V

card ①

1.  $x(x - 3)^2 = x$

2.  $x^4 - 5x^3 - 14x^2 = 0$

3.  $\frac{1}{3}x^4 - 16 = 11$

Level V

card ②