

*This mathematics lesson gives children insight into algorithms and the usefulness of a step-by-step plan: how you can save time.*

## Algorithms and the Hanoi Tower

In this lesson the children get to work with the Tower of Hanoi. What are the steps you can take to move the discs from the left to the right? And how could help a step-by-step plan?

### What is the learning goal of this lesson?

- The pupils learn that an algorithm is a list of steps in the right order (a recipe)
- The pupils learn that following a step-by-step plan produces time savings and a good answer
- The children learn the game “The Hanoi Tower

### What is an algorithm?

Put butter in the pan first and then the eggs. An algorithm is also a recipe, but then for mathematicians and computer scientists. If you take all steps in the right order, it won't go wrong. But it could be sometimes faster.

### The Hanoi Tower

The French mathematician **Édouard Lucas** invented the game of the **Hanoi Tower** in 1883. He describes a legend about a Hindu temple in the Indian city of Benares under Emperor Fo Hi. The Brahmins, priests of this temple, are busy moving a tower of 64 gold discs. The discs lie on three needles of diamond, one el long (about 69 cm) and as thick as the body of a bee. The priests say that the world comes to an end when the work is finished.

If the priests were to move one disk per second, it would take  $2^{64} - 1$ , about  $1.84 \times 10^{19}$  seconds to move the tower. This is about 585 billion years. Compared to the predicted age of the sun (another 5 million years to go) this is very long

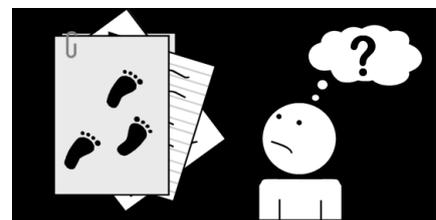
### The game

Initially, the question should be as open as possible: How can you transfer the disks as quickly as possible from one side to the other? The teacher gives the students the freedom to come up with a winning strategy.

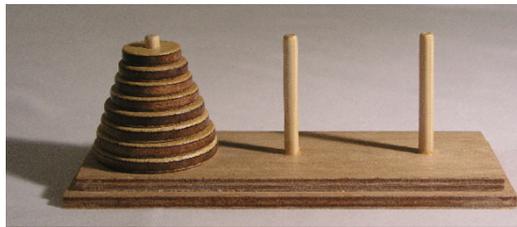
There are two rules:

- The smallest disc must always be at the top, so when moving, a large drive may not end up on a smaller one.
- And only one disk may be moved at a time.

First the students try it themselves, only after that some strategies are offered. While trying, mistakes are easily made, which means that it takes longer before the tower is moved. It is also important to bring the discs to the other side as quickly as possible. This stage of



messing around is important to involve pupils in a playful way with the problem. It is also possible that a pupil himself comes up with one of the strategies. Let the students record the time they spend on it.



### Solving methods

After the students themselves have first manually moved the disks, the teacher offers methods to solve the puzzle systematically. Here, too, the pupil should record how long they spend on a solution.

#### First move:

(0) If there is an odd number of disks, place the first disk on the stick on which you want to end up. If there is an even number of disks, place the first disk on the other stick.



Method 1: Take turns every other turn until the solution is found:

(1) Move the disk that is not the smallest according to the rules (there is only one possibility for this now).

(2) Move the smallest drive to the stick where it did not come from most recently. In other words, always move the smallest disk from stick 1 to stick 2, from 2 to 3, from 3 to 1 (or always the other way round).

#### Method 2:

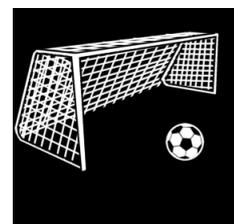
(1) Give the discs a different color.

(2) Never move the same disc twice.

(3) Never place a disc on a disc that has the same color.

#### Task:

Compare the times, which method (manually trying, method 1 or method 2) took the least time? From this draw a conclusion about step-by-step plans.



#### Sources:

Dijkgraag, Robbert, Louise Fresco & Bas Haring. De Bètacanon Junior. [www.juniorbetacanon.nl](http://www.juniorbetacanon.nl)

Wikipedia.org Torens van Hanoi. [https://en.wikipedia.org/wiki/Tower\\_of\\_Hanoi](https://en.wikipedia.org/wiki/Tower_of_Hanoi)